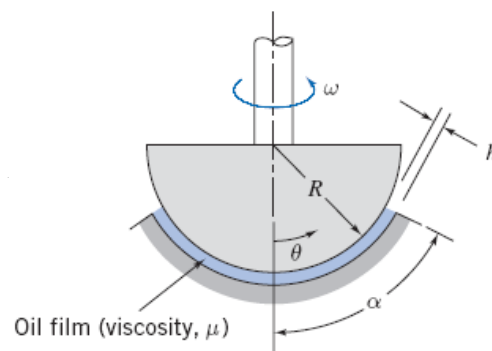


## Problem 2.75

[Difficulty: 5]

**2.75** A spherical thrust bearing is shown. The gap between the spherical member and the housing is of constant width  $h$ . Obtain and plot an algebraic expression for the nondimensional torque on the spherical member, as a function of angle  $\alpha$ .



Solution: Apply definitions

Computing equations:  $\tau = \mu \frac{du}{dy}$        $T = \int_A r \tau dA$

Assumptions: (1) Newtonian fluid, (2) narrow gap, (3) laminar flow

From the figure,  $r = R \sin \theta$        $u = \omega r = \omega R \sin \theta$

$$\tau = \mu \frac{du}{dy} = \mu \left( \frac{u - 0}{h} \right) = \mu \frac{u}{h} = \mu \frac{\omega R \sin \theta}{h}$$

$$dA = 2\pi r R d\theta = 2\pi R^2 \sin \theta d\theta$$

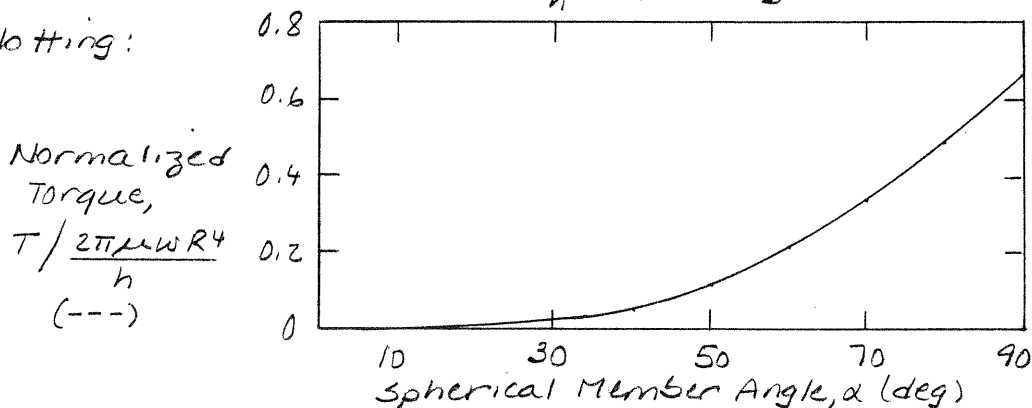
Thus

$$T = \int_0^\alpha R \sin \theta \left( \frac{\mu \omega R \sin \theta}{h} \right) 2\pi R^2 \sin \theta d\theta = \frac{2\pi \mu \omega R^4}{h} \int_0^\alpha \sin^3 \theta d\theta$$

$$T = \frac{2\pi \mu \omega R^4}{h} \left[ \frac{\cos^3 \theta}{3} - \cos \theta \right]_0^\alpha = \frac{2\pi \mu \omega R^4}{h} \left[ \frac{\cos^3 \alpha}{3} - \cos \alpha + \frac{2}{3} \right]$$

To plot, normalize to  $\left[ T / \frac{2\pi \mu \omega R^4}{h} \right] = \left[ \frac{\cos^3 \alpha}{3} - \cos \alpha + \frac{2}{3} \right]$

Plotting:



{ Check dimensions:  $\left[ \frac{\mu \omega R^4}{h} \right] = \frac{Ft}{L^2} \times \frac{1}{t} \times L^4 \times \frac{1}{L} = FL \checkmark \checkmark$  }